

Generalized Quantum Potentials with Hypergeometric Eigenfunctions*.

J. J. Peña^{1}**, J. García-Martínez², J. García-Ravelo² and J. Morales¹.

¹Universidad Autónoma Metropolitana – Azc.
DCBI – Departamento de Ciencias Básicas, Área de Física Atómica
Molecular Aplicada, San Pablo 180, 02200 México, D. F.

²Instituto Politécnico Nacional, Escuela Superior de Física y Matemáticas,
Departamento de Física, División de Posgrado, Edificio 9
Unidad Profesional Adolfo López Mateos, 07738 México, D.F.

**). Corresponding author: jjpg@correo.azc.uam.mx

A canonical $g(x)$ transformation method to convert a general second order differential equation (DE) into a Schrödinger-like DE is presented. As a useful application of the proposal we consider explicitly the hypergeometric DE in order to find the exactly-solvable quantum potentials having hypergeometric wavefunctions. As a result, it appears that different exactly-solvable potentials can be obtained depending on the choice of the $g(x)$ involved transformation. Specifically, the generalized Scarf, Posh-Teller, Eckart and Rosen-Morse trigonometric and hyperbolic potentials, are obtained by selecting $g(x)$ as constant and proportional to the $P(x)$ hypergeometric coefficient. Similarly, the choice $g(x) \sim P(x)/x^2$ and $g(x) \sim x^2/P(x)$ leads to the exactly-solvable generalized multiparameter exponential-type potential which contain as particular cases the Hulthén, Manning-Rosen and Woods-Saxon potentials, among others. Moreover, the proposed transformation method is general and can be straightforwardly used not only to unify the study of exactly-solvable quantum potentials with special functions solutions but also as a method to construct new potentials, provided that it can be applied to other different DE, that could be useful in quantum chemical calculations.

*). Work partially supported by projects UAM-AZC-CBI-232004-09, COFFA-SIP-IPN-20110242 and SNI-Conacyt, MEXICO.